Bayes Days 2000 at LANL

Three-Day Minicourse on Bayesian Analysis in Physics

Lectures presented by

Prof. Volker Dose
Max Planck Institute for Plasma Physics

Sponsored by
Enhanced Surveillance Program, Los Alamos National Laboratory

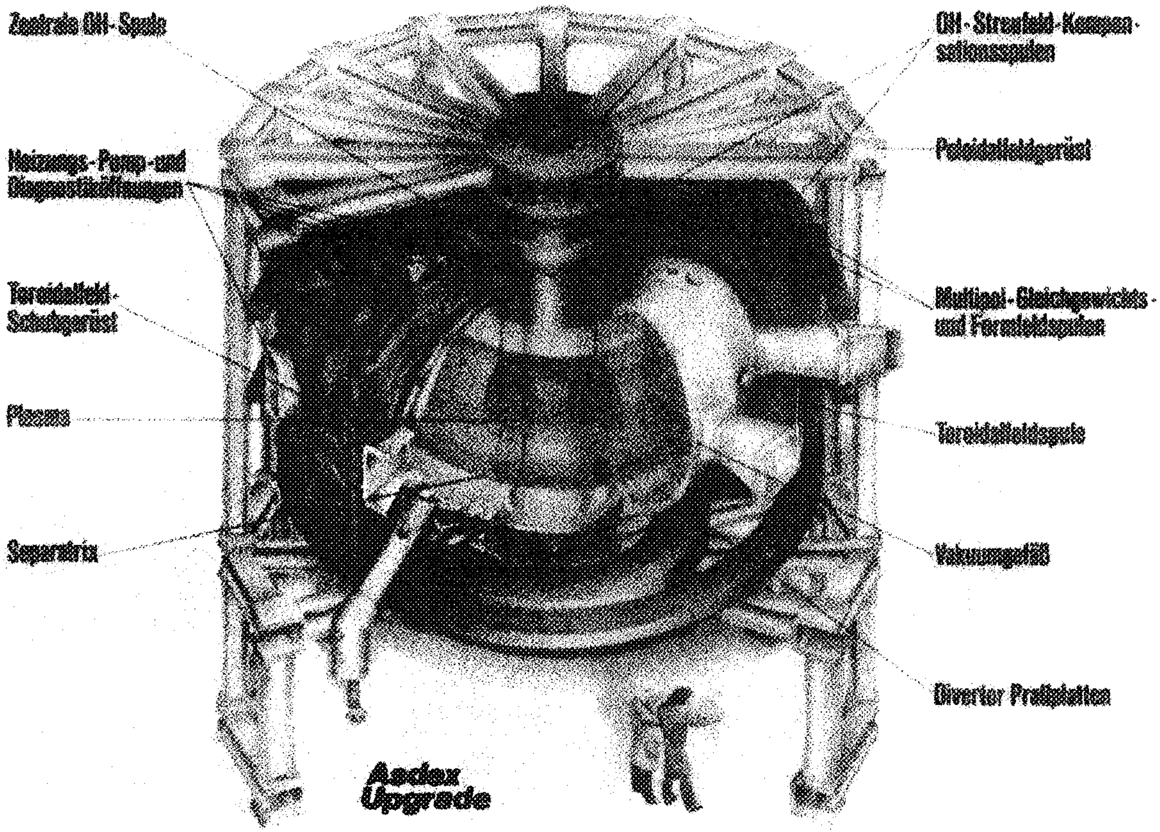
For more information, look on the web:
http://public.lanl.gov/kmh/course/BD2000.html

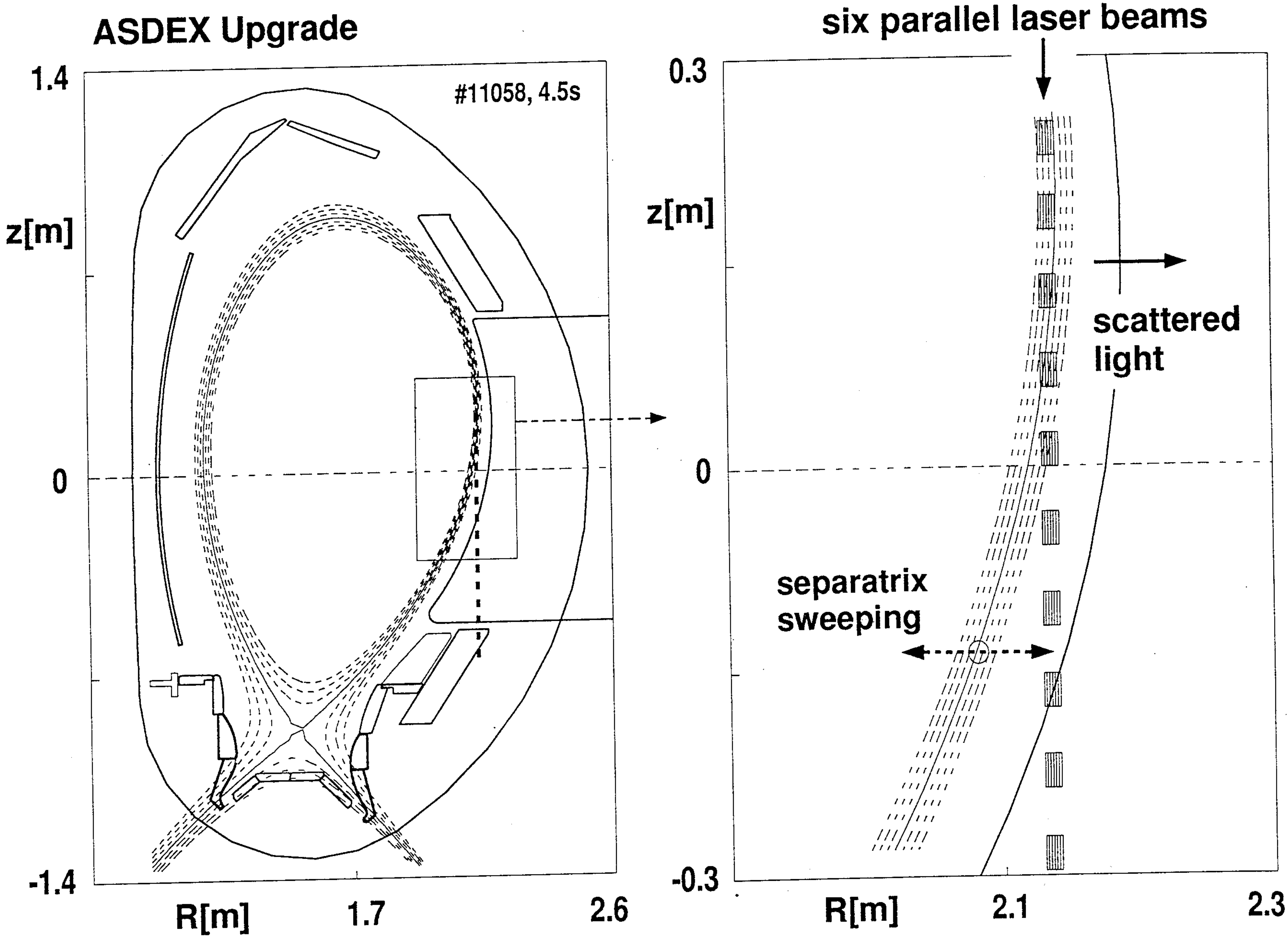
Organized by Ken Hanson, DX-3, 505-667-1402, kmh@lanl.gov

Fitting scattered data

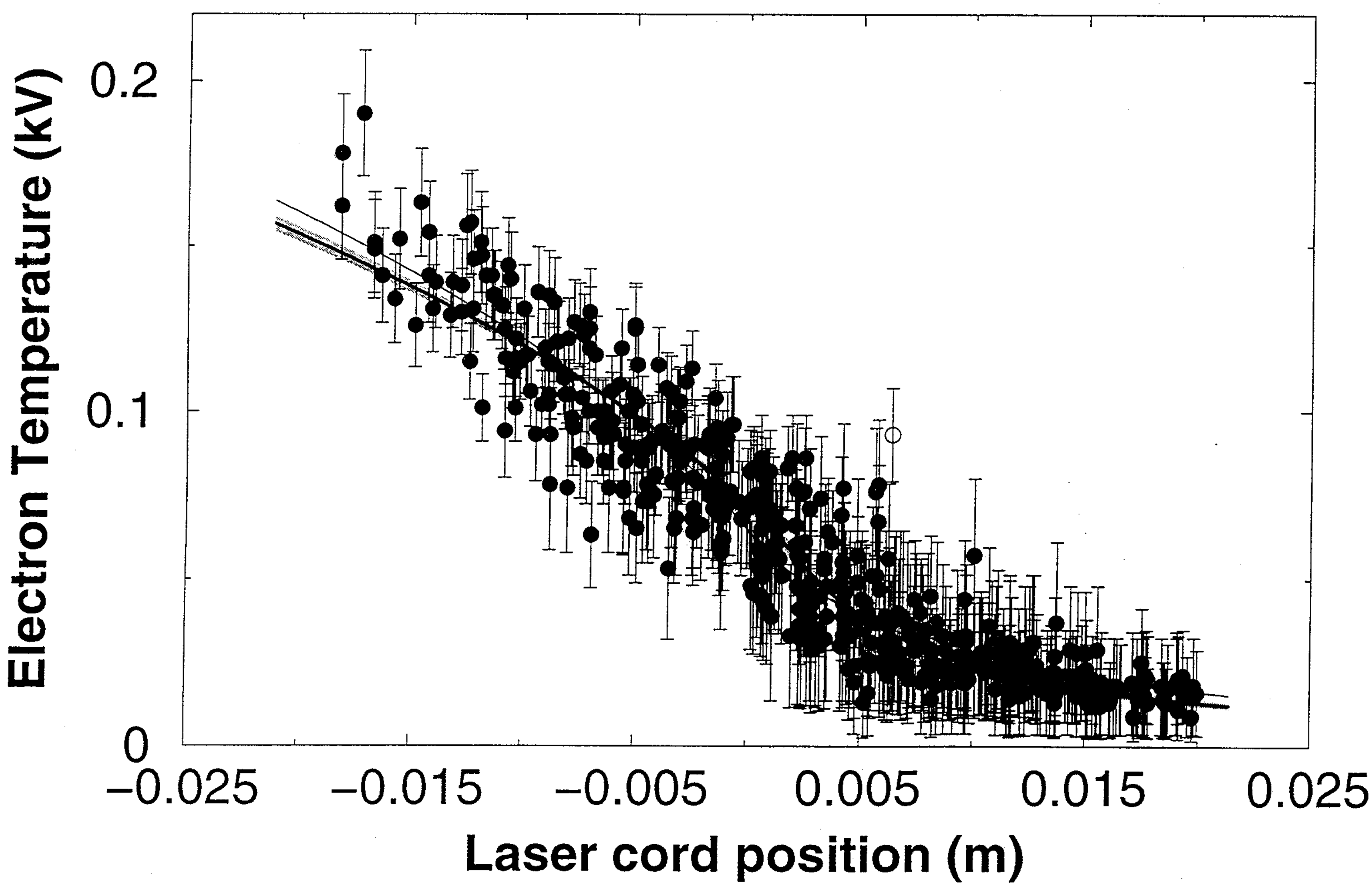
V. Dose and J. Neuhauser

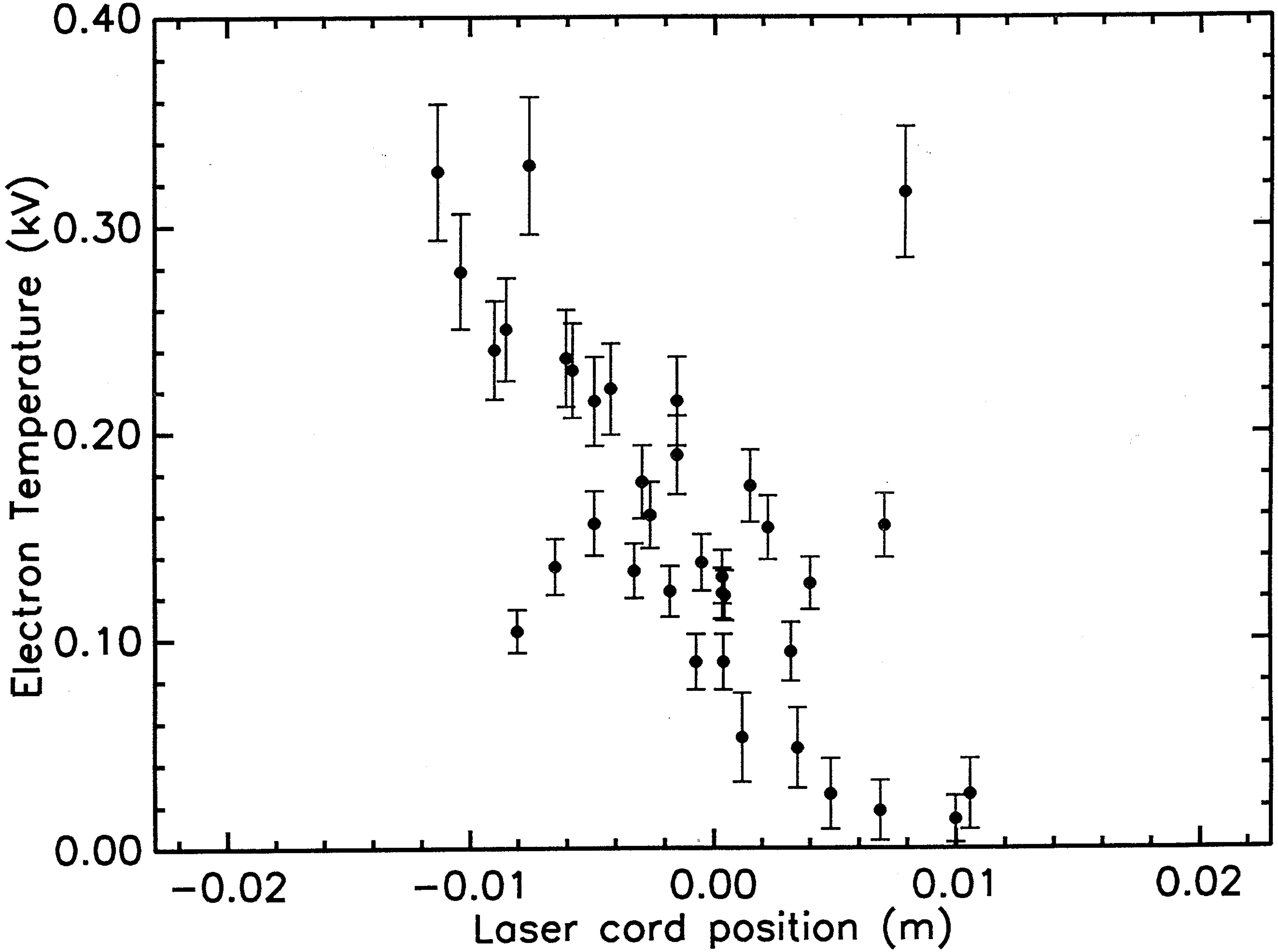
Los Alamos Nat. Lab
April 3 – 5, 2000


Zentrale OH-Spule
Poloidalfeldgerüst
Plasma
Toroidalfeldspule
Separatrix
Vakuumgefäß
Divertor Prallplatten
Asdex Upgrade



ASDEX Upgrade
#11058, 4.5s
1.4
z[m]
0
-1.4
R[m]
1.7
2.6
six parallel laser beams
0.3
z[m]
0
-0.3
scattered
light
separatrix
sweeping
R[m]
2.1
2.3


Electron Temperature (kV)
0.2
0.1
0
−0.025
−0.015
−0.005
0.005
0.015
0.025
Laser cord position (m)


0.40
0.30
0.20
0.10
0.00
Electron Temperature (kV)
−0.02
−0.01
0.00
0.01
0.02
Laser cord position (m)

model functions

$$\left.\begin{array}{ll} T(x) = \frac{c}{\lambda^{2/7}}\left(1+\frac{x-x_0}{\lambda}\right)^{-\frac{4}{5}} & x \geq x_0 \\ T(x) = \frac{c}{\lambda^{2/7}}\left(1-\frac{4}{5}\frac{x-x_0}{\lambda}\right) & x \leq x_0 \end{array}\right\} \begin{array}{c} \text{const} \\ \text{head cond.} \end{array}$$

$$\left.\begin{array}{ll} T(x) = \frac{c}{\lambda^{2/7}}\left(1+\frac{x-x_0}{\lambda}\right)^{-\frac{4}{3}} & x \geq x_0 \\ T(x) = \frac{c}{\lambda^{2/7}}\left(1-\frac{8}{3}\frac{x-x_0}{\lambda}\right)^{\frac{1}{2}} & x \leq x_0 \end{array}\right\} \begin{array}{c} \text{head cond.} \\ \sim \mathrm{T} \end{array}$$

likelihood: "regular data"

$$d_i = T(x_i, c, x_0, \lambda) + \varepsilon_i$$

assume $\langle \varepsilon_i \rangle = 0$ and $\langle \varepsilon_i^2 \rangle = \rho_i^2$

$$p(d_i | x_i, c, x_0, \lambda, \rho_i) = \frac{1}{\rho_i \sqrt{2\pi}} \exp\left\{-\frac{(d_i - T_i)^2}{2\rho_i^2}\right\}$$

$\rho_i =$ true error, unknown, estimate σ_i:

$$p(\rho_i | \sigma_i) = \delta(\rho_i - \sigma_i)$$

likelihood: "irregular data"

$$d_i = T(x_i, c, x_0, \lambda) + \varepsilon_i \pm s_i$$

$$p^{\pm}(d_i | x_i, c, x_0, \lambda, \sigma_i, s_i, I) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{ -\frac{(d_i - T_i \pm s_i)^2}{2\sigma_i^2} \right\}$$

marginalize s_i

$$p^{\pm}(d_i | x_i, c, x_0, \lambda, \sigma_i, I) = \int p^{\pm}(d_i, s_i | x_i, c, x_0, \lambda, \sigma_i, I)\ ds_i$$

$$= \int p^{\pm}(s_i | I) \cdot p^{\pm}(d_i | x_i, c, x_0, \lambda, \sigma_i, s_i, I)\ ds_i$$

assume $\langle s_i \rangle = \xi$ for all i

$$p^{\pm}(s_i | \xi, I) = \frac{1}{Z} \exp\left\{ \mp \frac{s_i}{\xi} \right\}$$

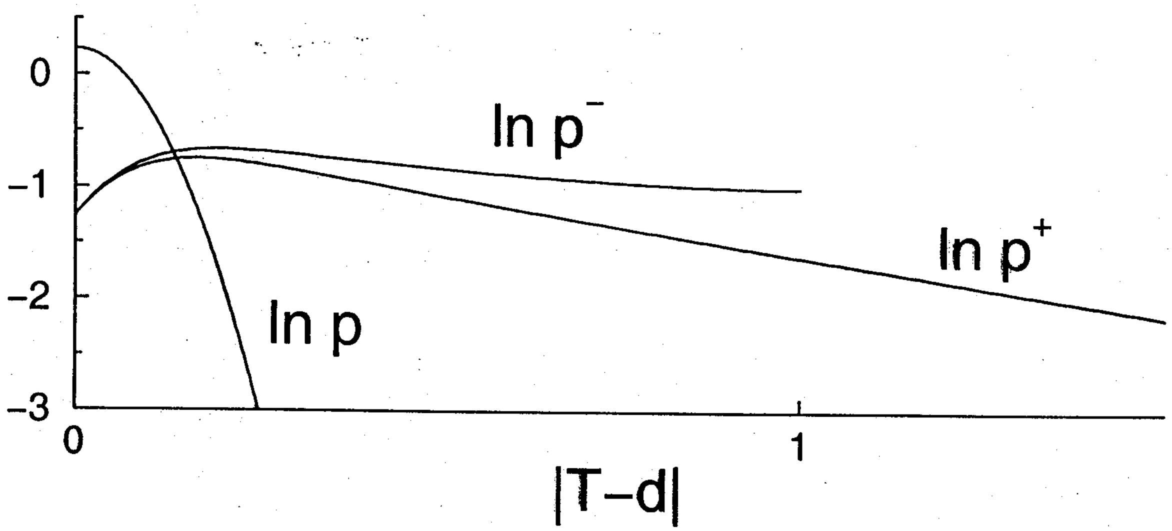
$+$ sign for $d > T$ in $0 \leq d - T < \infty$

$-$ sign for $d < T$ in $0 \leq T - d \leq T$

$$Z_i = \xi \left[2 - \exp\left(-\frac{T_i}{\xi} \right) \right]$$

$$p^{+}(d_i|x_i,\sigma_i,T_i,\xi,I) = \exp\left\{-\frac{d_i - T_i}{\xi} + \frac{\sigma_i^2}{2\xi^2}\right\}$$
$$\cdot\left\{\mathrm{erf}\left[(d_i + \frac{\sigma_i^2}{\xi})/\sigma_i\sqrt{2}\right] - \mathrm{erf}\left[(d_i - T_i + \frac{\sigma_i^2}{\xi})/\sigma_i\sqrt{2}\right]\right\}/Z$$

$$p^{-}(d_i|x_i,\sigma_i,T_i,\xi,I) = \exp\left\{\frac{d_i - T_i}{\xi} + \frac{\sigma_i^2}{2\xi^2}\right\}$$
$$\cdot\left\{1 + \mathrm{erf}\left[(d_i - T_i + \frac{\sigma_i^2}{\xi})/\sigma_i\sqrt{2}\right]\right\}/Z$$



Mixture model

$$p(\vec{d}|\vec{x},\vec{\sigma},\vec{T},\xi,I) = \prod_i (\beta A_i + (1-\beta)B_i)$$

Posterior expectations

$$p(c, x_0, \lambda, \beta|\vec{x}, \vec{d}, \vec{\sigma}, I) = \frac{p(c, x_0, \lambda, \beta|I)}{p(\vec{d}|\vec{x}, \vec{\sigma}, I)} p(\vec{d}|\vec{x}, \vec{\sigma}, c, x_0, \lambda, \beta, I)$$

$$p(c, x_0, \lambda, \beta|I) = p(c|I) \cdot p(x_0|I) \cdot p(\lambda|I) \cdot p(\beta|I)$$

$$p(\beta|I) = 1,\ 0 \le \beta \le 1,\ P(\lambda) = \text{flat, improper}$$

$$p(c|c_0, \Delta c, I) = \exp\left\{-\frac{1}{2}\left[\frac{c - c_0}{\Delta c}\right]^2\right\} / \Delta c\sqrt{2\pi}$$

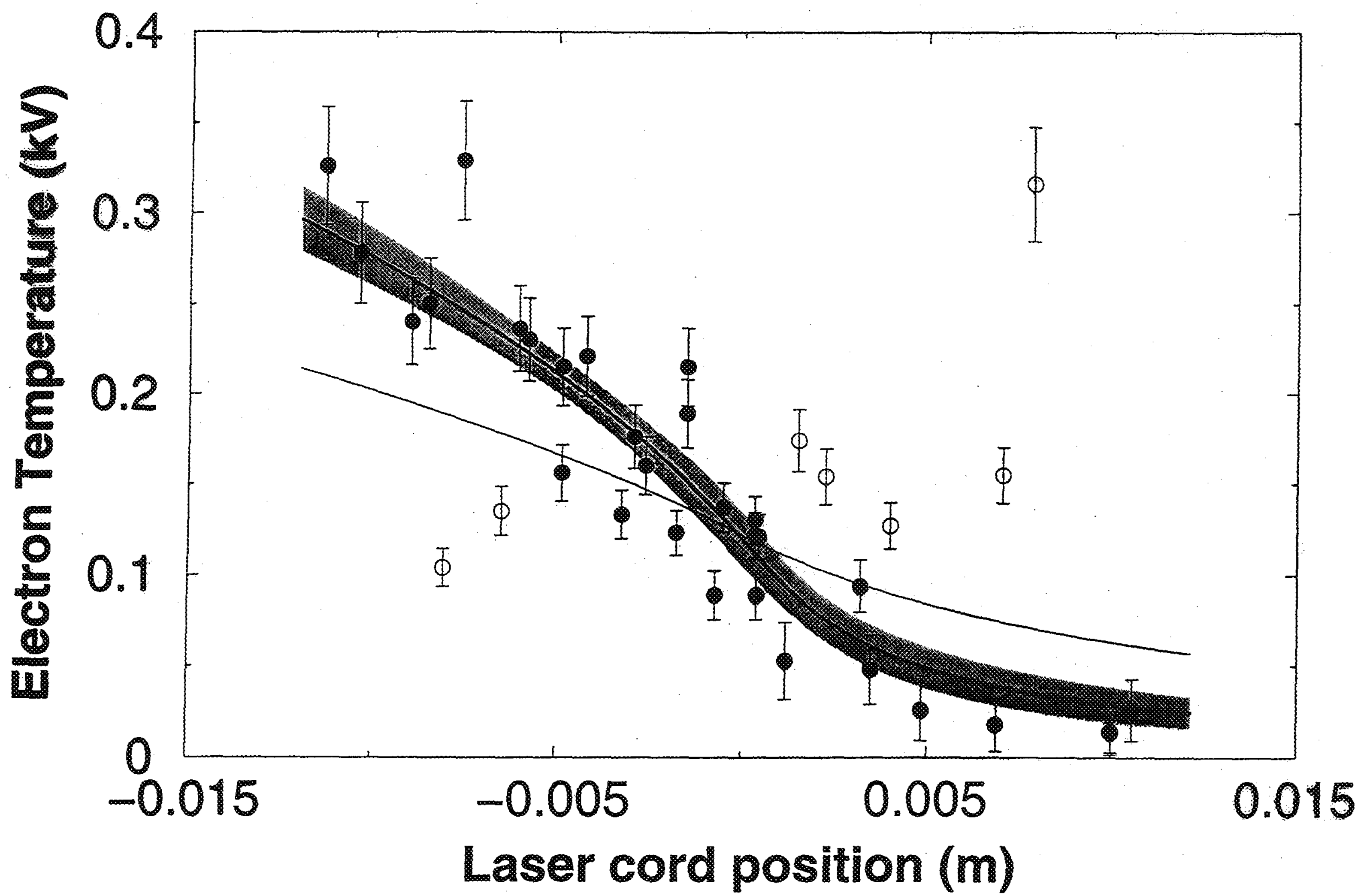
$$p(x_0|\Delta x_0, I) = \exp\left\{-\frac{1}{2}\left[\frac{x_0}{\Delta x_0}\right]^2\right\} / \Delta x_0\sqrt{2\pi}$$

Parameters

$$\langle \mu_k^n \rangle = \int dc \int dx_0 \int d\lambda \int d\beta\ \mu_k^n\ p(c, x_0, \lambda, \beta|\vec{x}, \vec{d}, \vec{\sigma}, I)$$

Function

$$\langle T^n(x) \rangle = \int [T(x, x_0, c, \lambda]^n \cdot p(c, x_0, \lambda, \beta|\vec{x}, \vec{d}, \sigma, I)\ dc dx_0 d\lambda d\beta$$


0.4
0.3
0.2
0.1
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−0.015
−0.005
0.005
0.015
Electron Temperature (kV)
Laser cord position (m)